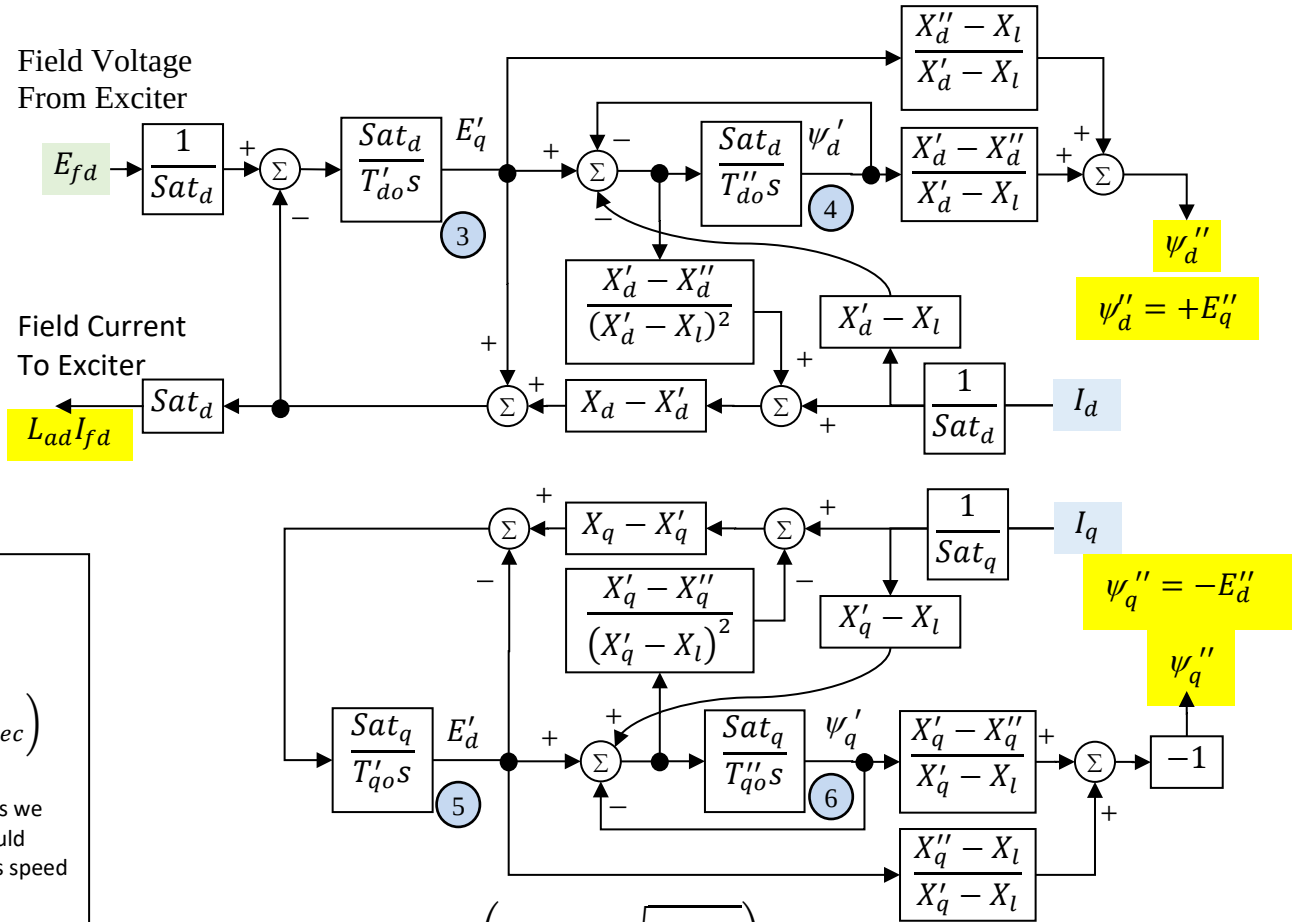
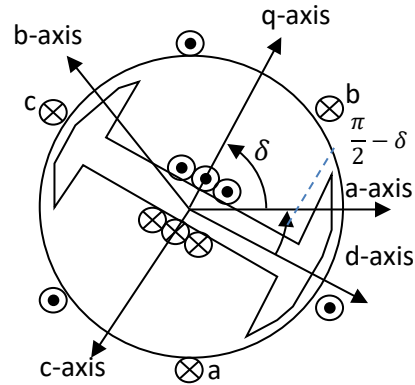


Machine Model GENTPW

Machine Model GENTPW



Mechanical Swing Equations

$$\begin{aligned} \delta &= \omega * \omega_0 \\ \dot{\omega} &= \frac{1}{2H} \left(\frac{P_{mech} - D\omega}{1 + \omega} - T_{elec} \right) \end{aligned}$$

ω = per unit speed deviation, so $\omega = 0$ means we are at synchronous speed and $\omega = 1$ would mean it's spinning at double synchronous speed
 ω_0 = synchronous speed $2\pi f_0$ where f_0 is the nominal system frequency in Hz

Note: If option *Ignore Speed Effects in Generator Swing Equation* is true, then instead use

$$\dot{\omega} = \frac{1}{2H} (P_{mech} - D\omega - T_{elec})$$

$$Sat_d = Sat_q = 1 + Sat \left(\psi_m + K_{is} \sqrt{I_d^2 + I_q^2} \right)$$

$$\psi_m = \frac{1}{1+\omega} \sqrt{(V_{qag})^2 + \left(\frac{X_d - X_l}{X_q - X_l} (V_{dag}) \right)^2}$$

$$V_{qag} = V_{qterm} + I_q R_a + I_d X_l$$

$$V_{dag} = V_{dterm} + I_d R_a - I_q X_l$$

The GENTPW model includes saturation not simply as an additive term to the derivative of the E'_d and E'_q terms, but instead as the saturation of all input parameters directly (impedance, time constants, field voltage, and field current). This means that the network interface equations and Electrical Torque equations are impacted by the saturation as well. The relationships are as described in the following.

Saturated Impedance Terms

$$Sat_d = Sat_q = 1 + Sat \left(\psi_m + K_{is} \sqrt{I_d^2 + I_q^2} \right)$$

$$\psi_m = \frac{1}{1+\omega} \sqrt{(V_{qag})^2 + \left(\frac{X_d - X_l}{X_q - X_l} \right) (V_{dag})^2} \quad (\text{call this this magnetizing flux})$$

$$V_{qag} = V_{qterm} + I_q R_a + I_d X_l \quad (\text{air gap voltage})$$

$$V_{dag} = V_{dterm} + I_d R_a - I_q X_l \quad (\text{air gap voltage})$$

$$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l$$

$$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

Electrical Torque

$$\psi_q = \psi_q'' - I_q X''_{qsat}$$

$$\psi_d = \psi_d'' - I_d X''_{dsat}$$

$$T_{elec} = \psi_d I_q - \psi_q I_d$$

Network Interface Equations

$$\mathbf{V} = \frac{d\mathbf{\Psi}}{dt} = j(1 + \omega) \left(\psi_d'' + j\psi_q'' \right)$$

$$V_d + jV_q = \left(-\psi_q'' + j\psi_d'' \right) (1 + \omega)$$

$$V_d + jV_q = \left(E_d'' + jE_q'' \right) (1 + \omega)$$

Because of sub-transient saliency ($X_d'' \neq X_q''$), X''_{dsat} may not be equal to X''_{qsat} . Because of this the network interface equations cannot be written as a complex number circuit equation. Instead the network equations are written as follows

$$V_{dterm} = V_d - R_a I_d + X''_{qsat} I_q$$

$$V_{qterm} = V_q - X''_{dsat} I_d - R_a I_q$$

These equations must be used directly when modeling the network boundary equations

Treatment of R_{comp} and X_{comp}

When specified, the compensated voltage fed as an input to the exciter is calculated as:

$$V_{comp} = |\bar{V}_t - (R_{comp} + jX_{comp})\bar{I}_t|$$

Solid Rotor Machine

Time Constants must be greater than zero

Reactances must be greater than zero

Salient Pole Machine with single amortisseur windings on d and q axis

(see block diagram in subsequent pages)

$$T'_{qo} = 0 ; X'_q = X_q ; T'_{do} > 0 ; T''_{qo} > 0 ; T''_{do} > 0$$

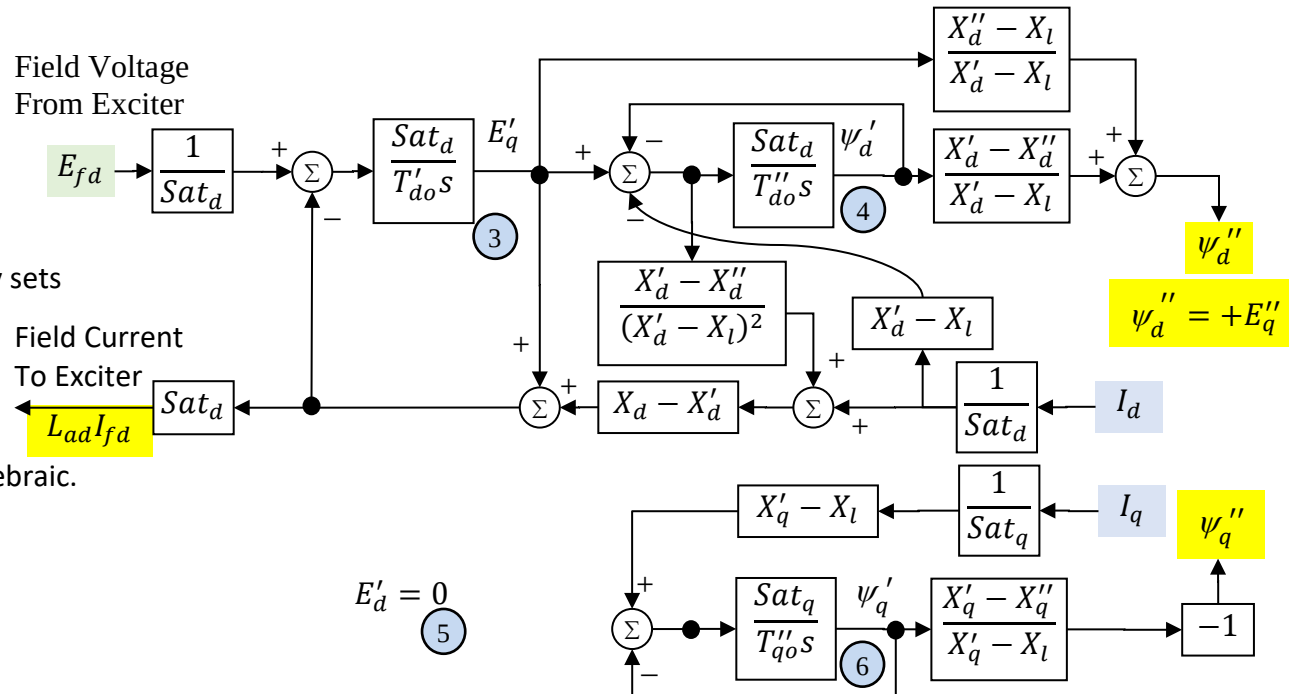
Salient Pole Machine without amortisseur windings

(see block diagram in subsequent pages)

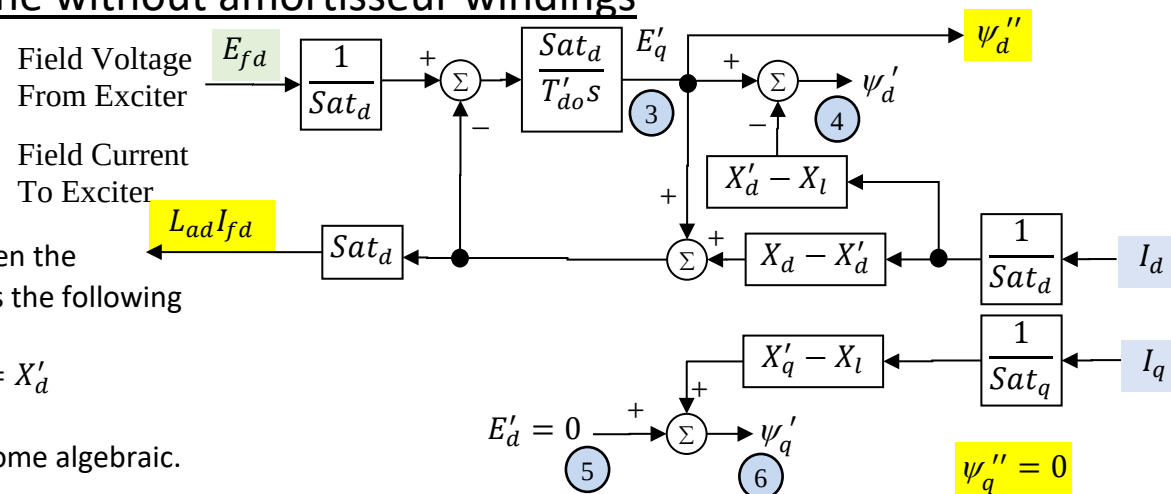
$$T'_{do} > 0 ; T'_{qo} = T''_{qo} = T''_{do} = 0 ; X''_d = X'_d ; X''_q = X'_q = X_q$$

$$\begin{aligned} T'_{qo} &= 0 \text{ and } X'_q = X_q \\ T'_{do} &> 0 \\ T''_{qo} &> 0 \\ T''_{do} &> 0 \end{aligned}$$

States E'_d become algebraic.


$$\begin{aligned} T'_{do} &> 0 \\ T'_{qo} &= T''_{qo} = T''_{do} = 0 \\ X''_d &= X'_d \\ X''_q &= X'_q = X_q \end{aligned}$$

States E'_d , ψ'_q and ψ'_d become algebraic.



Theory of model: Modification of Saturation Model used in GENROU

The improvements with the GENTPW are similar as for GENTPF and GENTPJ models and are primarily related to the treatment of saturation. It has however been noticed that the dynamic equations (block diagram) in GENTPF and GENTPJ are simplified from the GENROU resulting a small change in transient behavior. To avoid this, let's step back to the complete GENROU model and add a similar treatment as done for GENTPF for saturation.

Application of Saturation to the block diagram terms

To see where this comes from, let's go back to the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai and look at equations the derivation of the equations that led to the GENROU/GENSAL models. In particular, page 39 and 40 of that book and equations 3.120 and 3.130 – 3.139. A scanned image of these equations is shown in the follow image.

Orange terms do not saturate
Blue terms do saturate

$$X_d \triangleq \boxed{X_{\ell s}} + \boxed{X_{md}}, \quad X_q \triangleq \boxed{X_{\ell s}} + \boxed{X_{mq}} \quad (3.120)$$

$$X'_d \triangleq \boxed{X_{\ell s}} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} = X_{\ell s} + \frac{X_{md}X_{\ell fd}}{X_{fd}} = X_d - \frac{X_{md}^2}{X_{fd}} \quad (3.130)$$

$$X'_q \triangleq \boxed{X_{\ell s}} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} = X_{\ell s} + \frac{X_{mq}X_{\ell 1q}}{X_{1q}} = X_q - \frac{X_{mq}^2}{X_{1q}} \quad (3.131)$$

$$X''_d \triangleq \boxed{X_{\ell s}} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}} \quad (3.132)$$

$$X''_q \triangleq \boxed{X_{\ell s}} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}} + \frac{1}{X_{\ell 2q}}} \quad (3.133)$$

$$T'_{do} \triangleq \frac{\boxed{X_{fd}}}{\omega_s R_{fd}} \quad (3.134)$$

$$T'_{qo} \triangleq \frac{\boxed{X_{1q}}}{\omega_s R_{1q}} \quad (3.135)$$

$$T''_{do} \triangleq \frac{1}{\omega_s R_{1d}} \left(\boxed{X_{\ell 1d}} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} \right) \quad (3.136)$$

$$T''_{qo} \triangleq \frac{1}{\omega_s R_{2q}} \left(\boxed{X_{\ell 2q}} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} \right) \quad (3.137)$$

$$E_{fd} \triangleq \frac{\boxed{X_{md}}}{R_{fd}} V_{fd} \quad (3.139)$$

Taking a closer look at all the terms boxed in blue above, going back a page in the Sauer/Pai book are the following definitions.

$$X_{\ell s} \triangleq \frac{\omega_s \boxed{L_{\ell s}}}{Z_{BDQ}}, \quad X_{md} \triangleq \frac{\omega_s \boxed{L_{md}}}{Z_{BDQ}}, \quad X_{mq} \triangleq \frac{\omega_s \boxed{L_{mq}}}{Z_{BDQ}} \quad (3.115)$$

$$\boxed{X_{\ell fd}} \triangleq \boxed{X_{fd}} - \boxed{X_{md}}, \quad \boxed{X_{\ell 1d}} \triangleq \boxed{X_{1d}} - \boxed{X_{md}} \quad (3.118)$$

$$\boxed{X_{\ell 1q}} \triangleq \boxed{X_{1q}} - \boxed{X_{mq}}, \quad \boxed{X_{\ell 2q}} \triangleq \boxed{X_{2q}} - \boxed{X_{mq}} \quad (3.119)$$

These terms then refer to the green and ultimately red boxed terms in the following equations.

$$\boxed{X_{fd}} \triangleq \frac{\omega_s \boxed{L_{fdfd}}}{Z_{BFD}}, \quad \boxed{X_{1d}} \triangleq \frac{\omega_s \boxed{L_{1d1d}}}{Z_{B1D}}, \quad X_{fd1d} \triangleq \frac{\omega_s \boxed{L_{fd1d}} L_{sfd}}{Z_{BFD} L_{s1d}} \quad (3.116)$$

$$\boxed{X_{1q}} \triangleq \frac{\omega_s \boxed{L_{1q1q}}}{Z_{B1Q}}, \quad \boxed{X_{2q}} \triangleq \frac{\omega_s \boxed{L_{2q2q}}}{Z_{B2Q}}, \quad X_{1q2q} \triangleq \frac{\omega_s \boxed{L_{1q2q}} L_{s1q}}{Z_{B1Q} L_{s2q}} \quad (3.117)$$

$$\boxed{L_{md}} \triangleq \frac{3}{2}(L_A + L_B), \quad \boxed{L_{mq}} \triangleq \frac{3}{2}(L_A - L_B) \quad (3.105)$$

$$L_{rr}(\theta_{\text{shaft}}) \triangleq \begin{bmatrix} \boxed{L_{fdfd}} & \boxed{L_{fd1d}} & 0 & 0 \\ \boxed{L_{fd1d}} & \boxed{L_{1d1d}} & 0 & 0 \\ 0 & 0 & \boxed{L_{1q1q}} & \boxed{L_{1q2q}} \\ 0 & 0 & \boxed{L_{1q2q}} & \boxed{L_{2q2q}} \end{bmatrix} \quad (3.97)$$

Ultimately, all our reactances, time constants, and even the scaled field voltage and current end up being directly proportional to the following red-boxed terms.

- L_{md} : d-axis stator inductance
- L_{mq} : q-axis stator inductance
- L_{fdfd}, L_{1d1d} : d-axis rotor inductances
- L_{1q1q}, L_{2q2q} : q-axis rotor inductances

All of these six inductance terms represent inductance inside the windings of the synchronous machine. Thus all of these terms are going to experience saturation because their flux path is through the iron core. There is also dependence on the leakage inductance L_{ls} (boxed in purple above). The leakage inductance does not saturate because it is dominated by the characteristic of the air gap and we assume that the flux in the air does not saturate.

To derive the GENTPW model, assume all d-axis inductances are scaled by the Sat_d term and all q-axis inductances are scaled by the Sat_q term as follows.

$$\begin{aligned} L_{mdsat} &= \frac{L_{md}}{Sat_d} & L_{fdfdsat} &= \frac{L_{fdfd}}{Sat_d} & L_{1d1dsat} &= \frac{L_{1d1d}}{Sat_d} \\ L_{mqsat} &= \frac{L_{mq}}{Sat_q} & L_{1q1qsat} &= \frac{L_{1q1q}}{Sat_q} & L_{2q2qsat} &= \frac{L_{2q2q}}{Sat_q} \end{aligned}$$

This directly results in all the reactance terms being similarly scaled as follows.

$$\begin{aligned} X_{mdsat} &= \frac{X_{md}}{Sat_d} & X_{fdsat} &= \frac{X_{fd}}{Sat_d} & X_{1dsat} &= \frac{X_{1d}}{Sat_d} \\ X_{mqsat} &= \frac{X_{mq}}{Sat_q} & X_{1qsat} &= \frac{X_{1q}}{Sat_q} & X_{2qsat} &= \frac{X_{2q}}{Sat_q} \end{aligned}$$

As examples, the following carries this through calculating saturated values of the X_d'' and T_{do}'' terms. They are as follows and then expanded.

$$\begin{aligned} X_d'' &= X_l + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}} & T_{do}'' &= \frac{1}{\omega_s R_{1d}} \left(X_{\ell 1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} \right) \\ X_d'' &= X_l + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}} + \frac{1}{X_{1d} - X_{md}}} & T_{do}'' &= \frac{1}{\omega_s R_{1d}} \left(X_{1d} - X_{md} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}}} \right) \end{aligned}$$

Now apply saturation to the appropriate reactance terms and then simplify.

$$X''_{dsat} = X_l + \frac{1}{\frac{\frac{X_{md}}{Sat_d} + \frac{1}{\frac{X_{fd}}{Sat_d} - \frac{X_{md}}{Sat_d}} + \frac{1}{\frac{X_{1d}}{Sat_d} - \frac{X_{md}}{Sat_d}}}}$$

$$T''_{dosat} = \frac{1}{\omega_s R_{1d}} \left(\frac{X_{1d}}{Sat_d} - \frac{X_{md}}{Sat_d} + \frac{1}{\frac{\frac{X_{md}}{Sat_d} + \frac{1}{\frac{X_{fd}}{Sat_d} - \frac{X_{md}}{Sat_d}}}} \right)$$

$$X''_{dsat} = X_l + \frac{1}{Sat_d} \left(\frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}} + \frac{1}{X_{1d} - X_{md}}} \right)$$

$$T''_{dosat} = \frac{1}{Sat_d} \left[\frac{1}{\omega_s R_{1d}} \left(X_{1d} - X_{md} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}}} \right) \right]$$

$$X''_{dsat} = X_l + \frac{1}{Sat_d} \left(\frac{X''_d - X_l}{X''_d - X_l} \right)$$

$$T''_{dosat} = \frac{T''_{do}}{Sat_d}$$

This can be expressed more simply without all the intermediate terms as

$$X''_{dsat} = X_l + \frac{(X''_d - X_l)}{Sat_d}$$

$$T''_{dosat} = \frac{T''_{do}}{Sat_d}$$

You can go through the same procedure to write all the saturated reactance terms which have the same form as above, as well as the saturated time constants. Also notice in equation 3.139 of the Sauer/Pai book that the normalization of E_{fd} is proportional to X_{md} which will saturate. Similarly, the scaled field current $L_{ad}I_{fd}$ is also be subject to saturation. This gives us the following saturated values.

$$X_{dsat} = \frac{X_d - X_l}{Sat_d} + X_l$$

$$X_{qsat} = \frac{X_q - X_l}{Sat_q} + X_l$$

$$X'_{dsat} = \frac{X'_d - X_l}{Sat_d} + X_l$$

$$X'_{qsat} = \frac{X'_q - X_l}{Sat_q} + X_l$$

$$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l$$

$$X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

$$T'_{dosat} = \frac{T'_{do}}{Sat_d}$$

$$T'_{dosat} = \frac{T'_{do}}{Sat_d}$$

$$T'_{qosat} = \frac{T'_{qo}}{Sat_d}$$

$$T'_{qosat} = \frac{T'_{qo}}{Sat_d}$$

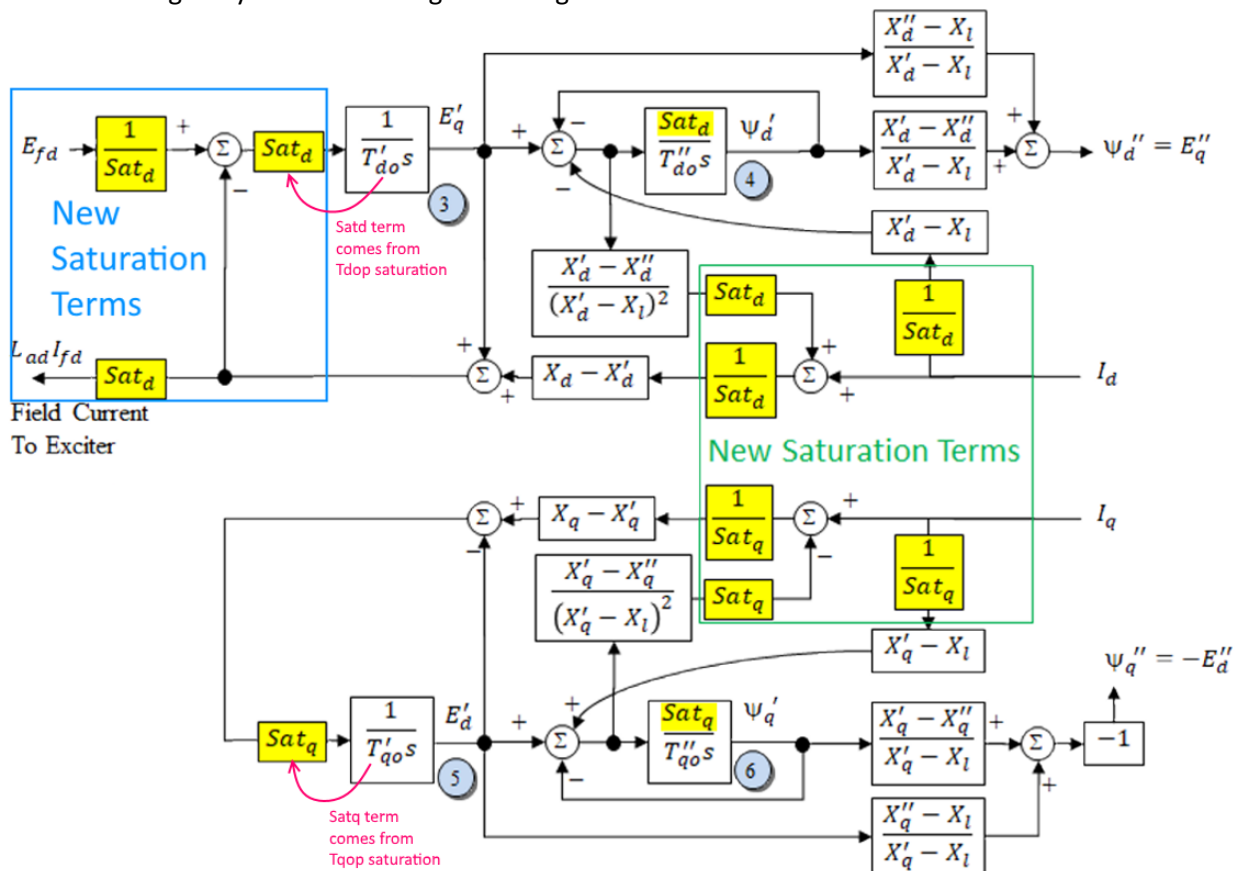
$$E_{fdsat} = \frac{E_{fd}}{Sat_d}$$

$$L_{ad}I_{fdsat} = Sat_d L_{ad}I_{fd}$$

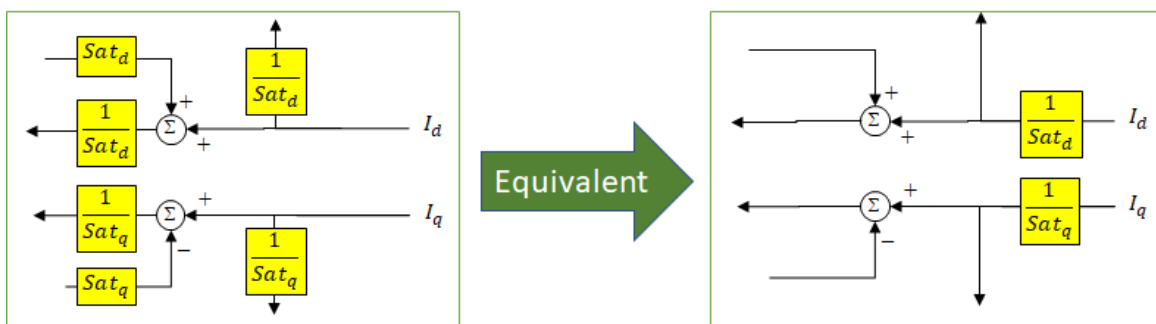
Note that the equations for X''_{dsat} and X''_{qsat} are identical to those which were used in the network boundary equations for GENPTF and GENTPJ. If we make these assumptions for network boundary equations of GENROU then we get the same treatment as GENTPF and GENTPJ. We can then also apply this to the block diagram for GENROU. There are 5 reactance blocks on both the d-axis and the q-axis portion of the GENROU block diagram. The following table shows how each block is translated to make the GENTPW dynamic equations block diagram.

Block on GENROU	Convert to Saturated Version	Intermediate Derivation Steps	Translates GENPTW
$X'_d - X_l$	$X'_{dsat} - X_l$	$\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - X_l$	$\frac{X'_d - X_l}{Sat_d}$
$X_d - X'_d$	$X_{dsat} - X'_{dsat}$	$\left[\frac{X_d - X_l}{Sat_d} + X_l \right] - \left[\frac{X'_d - X_l}{Sat_d} + X_l \right] = \frac{X_d - X_l}{Sat_d} - \frac{X'_d - X_l}{Sat_d}$	$\frac{X_d - X'_d}{Sat_d}$
$\frac{X'_d - X''_d}{X'_d - X_l}$	$\frac{X'_{dsat} - X''_{dsat}}{X'_{dsat} - X_l}$	$\frac{\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - \left[\frac{X''_d - X_l}{Sat_d} + X_l \right]}{\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - X_l} = \frac{\frac{X'_d - X''_d}{Sat_d}}{\frac{X'_d - X_l}{Sat_d}}$	$\frac{X'_d - X''_d}{X'_d - X_l}$
$\frac{X''_d - X_l}{X'_d - X_l}$	$\frac{X''_{dsat} - X_l}{X'_{dsat} - X_l}$	$\frac{\left[\frac{X''_d - X_l}{Sat_d} + X_l \right] - X_l}{\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - X_l} = \frac{\frac{X''_d - X_l}{Sat_d}}{\frac{X'_d - X_l}{Sat_d}}$	$\frac{X''_d - X_l}{X'_d - X_l}$
$\frac{X'_d - X''_d}{(X'_d - X_l)^2}$	$\frac{X'_{dsat} - X''_{dsat}}{(X'_{dsat} - X_l)^2}$	$\frac{\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - \left[\frac{X''_d - X_l}{Sat_d} + X_l \right]}{\left(\left[\frac{X'_d - X_l}{Sat_d} + X_l \right] - X_l \right)^2} = \frac{\frac{X'_d - X''_d}{Sat_d}}{\left(\frac{X'_d - X_l}{Sat_d} \right)^2}$	$\frac{X'_d - X''_d}{(X'_d - X_l)^2} Sat_d$

The exact same thing can be done on the q-axis and simply adds some multiplication or division by Sat_q instead. The conversion for the time constant and field voltage term are direct as they are just scaled by the saturation function. This gives you the following block diagram.



Doing a small amount of additional algebraic manipulation you can see that the new saturation terms in the image above can be simplified even further as shown in the following figure.



This then gives you the GENTPW block diagram from the first page of this document.

Looking at the block diagram, essentially what our simplification has done is change from scaling all the inductance terms to instead scaling the input stator currents. Think back to the basic relationship of flux equals inductance multiplied by current ($\psi = LI$). Our objective is to modify the resulting flux that is achieved, so what the algebra has done is simply changed the scaling from the inductance to the stator current instead.

Choosing saturation function input and scaling of d and q axis

To model saturation in the GENTPW model, as with all our machine models, only one saturation function is used. The saturation function is calculated from the open circuit test and is only testing the saturation on the d-axis. With the GENROU and GENROU and GENTPF/GENTPJ models the output of the saturation function applying it to the d-axis and then it scaled this value to apply it to the q-axis. The GENTPW model will handle this differently in the manner described in the following papers.

- L Pierrat, E Dejaeger, MS Garrido, "Models unification for the saturated synchronous machines," *International conference on Evolution and modern and Modern Aspects of Synchronous Machines*, 1991
- E. Levi, "State-space d-q axis models of saturated salient pole synchronous machines," in *IEEE Proceedings - Electric Power Applications*, vol. 145, no. 3, pp. 206-216, May 1998. doi: 10.1049/ip-epa:19981786
- F. Therrien, L. Wang, J. Jatskevich and O. Wasynczuk, "Efficient Explicit Representation of AC Machines Main Flux Saturation in State-Variable-Based Transient Simulation Packages," in *IEEE Transactions on Energy Conversion*, vol. 28, no. 2, pp. 380-393, June 2013.

For GENTPW, instead of scaling the output of the saturation function, the input from the q-axis is scaled instead. As described in the papers, the fundamental assumption is that the mutual inductance on the d and q axis will be scaled by the same factor. The assumption is that because you do not have any measured information about saturation on the q axis it is most appropriate to assume that they saturate to the same degree. The assumption is that they belong to the same magnetic body, so they will experience the same degree of saturation. This is actually the same assumption that was made for the GENROU model, however for GENROU saturation was introduced using an addition of flux instead of multiplying by a factor as GENTPW is doing. Because we were adding a flux we had to multiply by the factor $\frac{X_q - X_l}{X_d - X_l}$ so that the additional flux maintained the same scaling. In GENTPW we don't need the output scaling because we are introducing saturation as a multiplication term.

Choosing scaling for Magnetizing Flux

Stated mathematically these models are trying to enforce that $\frac{L_{mdsat}}{L_{mqsat}} = \frac{L_{md}}{L_{mq}}$. Remember that $X_{mdsat} = \frac{X_{md}}{Sat_d}$

and $X_{mqsat} = \frac{X_{mq}}{Sat_q}$, so this means that $\frac{X_{mdsat}}{X_{mqsat}} = \frac{X_{md}}{X_{mq}} \frac{Sat_q}{Sat_d}$. Thus in order for the assumptions that d and q axis saturation are the same requires that $Sat_d = Sat_q$. As a result the output of the saturation function will be applied directly to both the d and q axis with no scaling on the value going to the q-axis. The papers do however include a scaling on the input to the saturation function which is describe in detail in Section 2 of the Levi paper from 1998. The following is a brief overview of the derivation of the scaling term on the q-axis flux of $\sqrt{\frac{X_d - X_l}{X_q - X_l}}$.

The scaling is chosen because it results in a linear magnetic circuit. These models assume

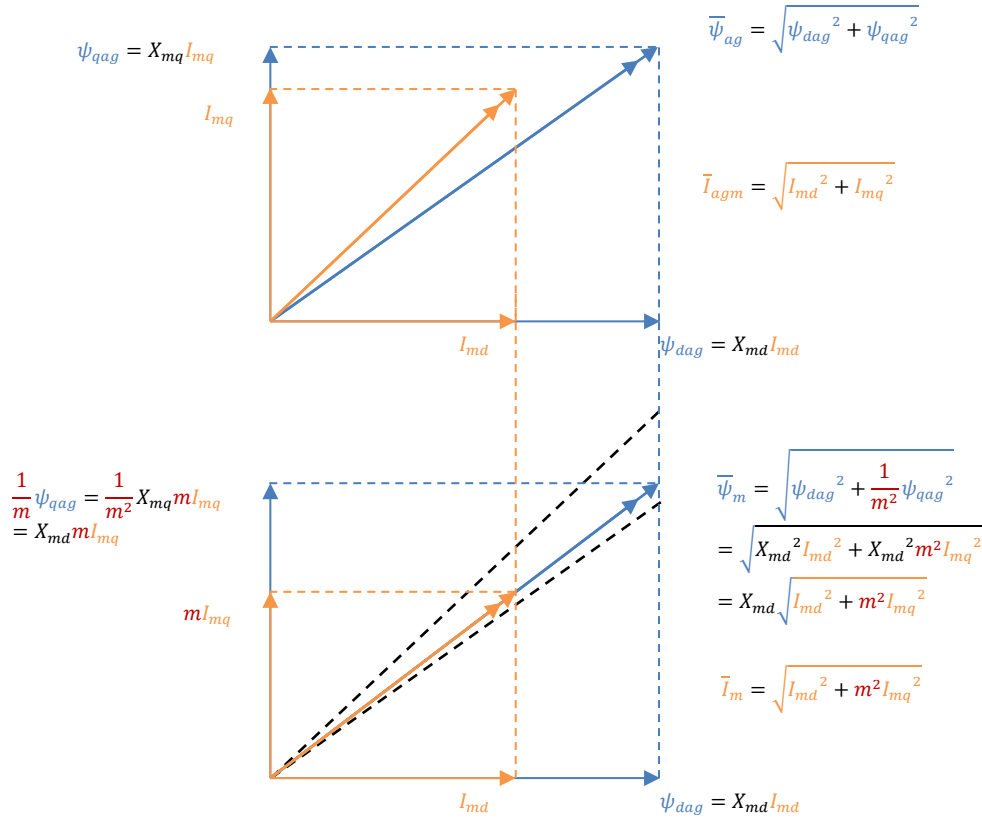
1. The stator windings (current can be transformed to the stator current on the d and q axis)
2. Two rotor windings on the d axis (including the field winding itself)
3. Two rotor windings on the q axis.

Then the total current is the summation of these currents and can be defined as the air gap magnetizing current:

$$\bar{I}_{agm} = I_{md} + jI_{mq} = (-I_d + I_{fd} + I_{1d}) + j(-I_q + I_{1q} + I_{2q})$$

We also define the air gap flux as a complex number $\bar{\psi}_{ag}$, but this is not used as an input to the saturation function. If we had chosen this, then $\bar{\psi}_{ag} = \psi_{dag} + j\psi_{qag} = X_{md}I_{md} + jX_{mq}I_{mq}$

Unfortunately, that would have meant that X_{md} cannot be factored out, which implies that the d-axis saturation curve cannot be directly used for both axes. Another way of stating the same thing is shown in the vector diagram below, since the direction of $\bar{I}_{agm}$ is not the same as $\bar{\psi}_{ag}$.



In order to suitably utilize ONLY the d-axis saturation curve for both axes, there is a trick that is used to ensure we get a linear magnetic circuit, such as $\bar{\psi}_m = X_{md} \bar{I}_m$. The Levi paper from 1998 shows this by defining a fictitious scaled magnetizing current $\bar{I}_m \triangleq I_{md} + jm I_{mq}$, which is a rotated version of $\bar{I}_{agm}$. By substituting, we get a scaled magnetization flux,

$$\begin{aligned}
 \bar{\psi}_m &= X_{md} (I_{md} + jm I_{mq}) \\
 &= X_{md} I_{md} + jm X_{md} I_{mq} \\
 &= X_{md} I_{md} + j \left[m^2 \frac{X_{md}}{X_{mq}} \right] \frac{1}{m} X_{mq} I_{mq} \\
 &= \psi_{dag} + j \left[m^2 \frac{X_{md}}{X_{mq}} \right] \frac{1}{m} \psi_{qag}
 \end{aligned}$$

As discussed in the Levi 1998 paper, the right choice for m is such that :

$$m^2 \frac{X_{md}}{X_{mq}} = 1$$

And then:

$$m = \sqrt{\frac{X_{mq}}{X_{md}}}$$

This leaves us with a scaled magnetizing flux, which is a rotated version of $\bar{\psi}_{ag}$:

$$\bar{\psi}_m \triangleq \psi_{dag} + j \frac{1}{m} \psi_{qag}$$

The outcome is the vector alignment of $\bar{\psi}_m$ and $\bar{I}_m$, and a linear magnetic circuit: $\bar{\psi}_m = X_{md} \bar{I}_m$. Hence, the d-axis saturation curve can be effectively used for predicting saturation on both axes.

Then simply remember that $X_{md} = X_d - X_l$ and $X_{mq} = X_q - X_l$, so we get our scaling on the input value for total magnetizing flux to be.

$$m = \sqrt{\frac{X_q - X_l}{X_d - X_l}}$$

Thus the magnetizing flux used as input to the saturation function is defined as

$$\bar{\psi}_m \triangleq \psi_{dag} + j \sqrt{\frac{X_d - X_l}{X_q - X_l}} \psi_{qag}$$

We then want the magnitude of this flux which is equal gives us the values

$$|\bar{\psi}_m| = \psi_m = \sqrt{\psi_{dag}^2 + \left(\frac{X_d - X_l}{X_q - X_l}\right) \psi_{qag}^2}$$

Getting the saturation function from the air gap voltage

In our simulations we are going to use the air gap voltage instead because it's readily calculated from the stator voltage and current using the complex circuit equations.

- $V_{qag} = V_{qterm} + I_q R_a + I_d X_l$
- $V_{dag} = V_{dterm} + I_d R_a - I_q X_l$

The air gap voltage however is equal to the derivative of the air gap flux, thus we have

- $V_{dag} + jV_{qag} = \frac{d(\psi_{dag} + j\psi_{qag})}{dt}$
- $V_{dag} + jV_{qag} = j(1 + \omega) (\psi_{dag} + j\psi_{qag})$ (multiply by $(1 + \omega)$ because these are sine waves)
- $V_{dag} + jV_{qag} = (1 + \omega) (-\psi_{qag} + j\psi_{dag})$

Therefore

- $\psi_{dag} = + \frac{V_{qag}}{(1 + \omega)}$
- $\psi_{qag} = - \frac{V_{dag}}{(1 + \omega)}$

From the air gap flux, the magnetizing flux is defined using a scaling on the d-axis air gap voltage as follows.

- $\psi_m = \sqrt{(\psi_{dag})^2 + \left(\frac{X_d - X_l}{X_q - X_l}\right) (\psi_{qag})^2}$
- $\psi_m = \frac{1}{1 + \omega} \sqrt{(V_{qag})^2 + \left(\frac{X_d - X_l}{X_q - X_l}\right) (V_{dag})^2}$

To simplify the conversion to the network reference frame during initialization, use some algebra to rewrite this

- $\frac{X_d - X_l}{X_q - X_l} = \frac{X_d - X_l}{X_q - X_l} - \frac{X_q - X_l}{X_q - X_l} + 1 = \frac{X_d - X_q}{X_q - X_l} + 1$

The magnetizing flux can then be rewritten as

- $\psi_m = \frac{1}{1 + \omega} \sqrt{(V_{qag})^2 + \left(\frac{X_d - X_q}{X_q - X_l} + 1\right) (V_{dag})^2}$
- $\psi_m = \frac{1}{1 + \omega} \sqrt{(V_{qag})^2 + (V_{dag})^2 + \left(\frac{X_d - X_q}{X_q - X_l}\right) (V_{dag})^2}$

We will also include the K_{is} to make the saturation a function of the stator current as well. Using all this the saturation function can be written as.

- $SatUsed = 1 + Sat\left(\psi_m + K_{is}\sqrt{I_d^2 + I_q^2}\right)$
- $SatUsed = 1 + Sat\left(\frac{1}{1+\omega}\sqrt{(V_{qag})^2 + (V_{dag})^2} + \left(\frac{X_d - X_q}{X_q - X_l}\right)(V_{dag})^2 + K_{is}\sqrt{I_d^2 + I_q^2}\right)$

Thus for GENTPW, the saturation function outputs are defined the same on the d and q axis and can be written as either of the following.

- $Sat_d = Sat_q = 1 + Sat\left(\frac{1}{1+\omega}\sqrt{(V_{qag})^2 + (V_{dag})^2} + \left(\frac{X_d - X_q}{X_q - X_l}\right)(V_{dag})^2 + K_{is}\sqrt{I_d^2 + I_q^2}\right)$
- $Sat_d = Sat_q = 1 + Sat\left(\frac{1}{1+\omega}\sqrt{(V_{qag})^2 + \left(\frac{X_d - X_l}{X_q - X_l}\right)(V_{dag})^2} + K_{is}\sqrt{I_d^2 + I_q^2}\right)$